

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

Marks

- 1 a) Test the convergence of $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ (2)
- b) Discuss the convergence of $\sum_{k=1}^{\infty} \frac{(2k)!}{4^k}$ (3)
- 2 a) Find the slope of the surface $z = \sin(y^2 - 4x)$ in the x - direction at the point $(3, 1)$. (2)
- b) Find the differential dz of the function $z = \tan^{-1}(x^2y)$. (3)
- 3 a) Find the direction in which the function $f(x, y) = xe^{xy}$ decreases fastest at the point $(2, 0)$. (2)
- b) Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at $(1, 1, 3)$ (3)
- 4 a) Evaluate $\iint_R y \sin xy \, dA$, where $R = [1, 2] \times [0, \pi]$. (2)
- b) Evaluate $\int_0^2 \int_0^1 \frac{x}{(1+xy)^2} \, dy \, dx$ (3)
- 5 a) if $\vec{A} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$, evaluate $\int \vec{A} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ (2)
along the path, $x = t, y = t^2, z = t^3$
- b) Prove that $\vec{F} = (x^2 - yz)\mathbf{i} + (y^2 - xz)\mathbf{j} + (z^2 - xy)\mathbf{k}$ is irrotational. (3)
- 6 a) Determine the source and sink of the vector field (2)
 $\vec{F}(x, y, z) = 2(x^3 - 2x)\mathbf{i} + 2(y^3 - 2y)\mathbf{j} + 2(z^3 - 2z)\mathbf{k}$
- b) Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$ where S is the surface of the cylinder $x^2 + y^2 = 4, z = 0$, (3)
 $z = 3$ where $\vec{F} = (2x - y)\mathbf{i} + (2y - z)\mathbf{j} + z^2\mathbf{k}$

PART B

Module 1

Answer any two questions, each carries 5 marks.

- 7 Check the convergence of the series $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \frac{3.4.5.6}{4.6.8.10} + \dots$ (5)
- 8 Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \frac{(-1)^k (x-4)^k}{3^k}$ (5)
- 9 Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^{2k-1}}{k^2+1}$ is absolutely (5)

convergent.

Module 1I

Answer any two questions, each carries 5 marks.

- 10 If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$ (5)
- 11 Let $z = xye^{\frac{x}{y}}$, $x = r \cos \theta$, $y = r \sin \theta$, use chain rule to evaluate $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ at $r = 2$ and $\theta = \frac{\pi}{6}$ (5)
- 12 A rectangular box open at the top is to have volume $32m^3$. Find the dimensions of the box requiring least material for its construction. (5)

Module III

Answer any two questions, each carries 5 marks.

- 13 Suppose that a particle moves along a circular helix in 3-space so that its position vector at time t is $\mathbf{r}(t) = 4\cos \pi t \mathbf{i} + 4\sin \pi t \mathbf{j} + t \mathbf{k}$. Find the distance travelled and the displacement of the particle during the time interval $1 \leq t \leq 5$. (5)
- 14 Suppose that the position vector of a particle moving in a plane $\mathbf{r} = 12\sqrt{t} \mathbf{i} + t^{\frac{5}{2}} \mathbf{j}$, $t > 0$. Find the minimum speed of the particle and locate when it has minimum speed? (5)
- 15 Find the parametric equation of the tangent line to the curve $x = \cos t$, $y = \sin t$, $z = t$ where $t = t_0$ and use this result to find the parametric equation of the tangent line to the point where $t = \pi$. (5)

Module 1V

Answer any two questions, each carries 5 marks.

- 16 Evaluate $\iint_R y \, dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$. (5)
- 17 Evaluate $\int_1^2 \int_0^x \frac{dy \, dx}{x^2 + y^2}$ (5)
- 18 Evaluate $\iiint_V x \, dx \, dy \, dz$ where V is the volume of the tetrahedron bounded by the plane $x = 0$, $y = 0$, $z = 0$ $x + y + z = a$. (5)

Module V

Answer any three questions, each carries 5 marks.

- 19 Find the scalar potential of $\vec{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ (5)
- 20 Find the work done by $F(x, y) = (x^2 + y^2)\mathbf{i} - x\mathbf{j}$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1, 0)$ to $(0, 1)$. (5)

- 21 Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y^2\vec{i} + xy\vec{j}$ and $\vec{r}(t) = t\vec{i} + 2t\vec{j}$, $1 \leq t \leq 3$. (5)
- 22 Evaluate $\int y dx + z dy + x dz$ along the path $x = \cos \pi t, y = \sin \pi t, z = t$ from $(1,0,0)$ to $(-1,0,1)$ (5)
- 23 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = \|\vec{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Using Stoke's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$; where $\vec{F} = xy\vec{i} + yz\vec{j} + xz\vec{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$ and $(0,0,1)$ in the first octant (5)
- 25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x)dy$ where C is the circle $x^2 + y^2 = 1$ (5)
- 26 Find the mass of the lamina that is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ if the density is $\rho(x, y, z) = x^2 z$. (5)
- 27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ across the surface σ bounded by $x^2 + y^2 = 4, z = 0$ and $z = 4$. (5)
- 28 If S is the surface of the sphere $x^2 + y^2 + z^2 = 1$, Evaluate $\iint_S (xi + 2yj + 3zk) \cdot dS$ (5)
